PA-003-1163002

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Seat No. ______

M. Sc. (Sem. III) (CBCS) Examination

May / June - 2018

CMT-3002 : Mathematics

(Functional Analysis) (New Course)

Faculty Code : 003
Subject Code : 1163002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are 5 questions.
(2) Attempt all the questions.

1 Answer any **seven** : (Each question carries 2 marks) **14**

(1) Define :

(1) Hilbert Space

(2) Banach Space.

(2) Give an example of a space that is not Banach space over K.

(3) Define :

(1) weak convergence in a n.l. space

(2) First Category.

(4) Is Dual of a Hilbert space is a Hilbert space? Justify your answer.

(5) State Zorn's lemma and Define sub linear functional.

(6) Is Every Separable Hilbert space is isomorphic to l^2 ? Justify your answer.

(7) State :

(1) Uniform Boundedness Principle.

(2) Hahn Banach Theorem.

(8) Define: Hyper plane and Hyperspace.

(9) Give an example of a norm which is not induced by any Inner Product Space.

(10) For a Norm linear space X over K and $\{x_n\} \subset X$ is $x_n \overrightarrow{w} x$ and $x_n \overrightarrow{w} y$ then $x = y$? Justify your answer.

2 Answer the following : **14**

(a) State without proof Open Mapping Theorem. Also prove that any two norms on a finite dimensional vector space X over K are equivalent.

OR

(a) State without proof Baire's theorem. Also prove that a Banach space cannot have a countably infinite Hamel basis.

(b) For a n.l. space X over K, prove that the dual space X' is separable $\Rightarrow$ X is separable.

3 Answer the following : **14**

(a) Let X, Y be a n.l. space over IK and $\|\cdot\|$ be the norm on $B(X,Y)$ defined by

$$\|T\| = \inf\left\{c > 0 / \|Tx\| \le c\|x\|, \forall\ x \in X\right\}$$

then prove that

$$\|T\| = \sup\left\{\frac{\|Tx\|}{\|x\|}, 0 \ne x \in X\right\} = \sup\left\{\|Tx\|, \|x\| = 1\right\}$$

(b) Give an example to show that a metric on a vector space X need not be induced by a norm on. Also provide arguments in support of your claim.

OR

(b) Define Canonical mapping C from a n.l. space X to X''. Also prove that $C : X \to X''$ is an isometry.

4 Answer the following : **14**

(a) State and prove Minkowski's Inequality.

(b) Prove that if every absolutely convergent series in $(X, \|\cdot\|)$ converges in $(X, \|\cdot\|)$ then $(X, \|\cdot\|)$ is a Banach Space over K.

5 Answer any **two** : **14**

(a) State only the projection theorem and if H is a Hilbert space and M is a non empty subset of H then prove that $\overline{span\,M} = M^{\perp\perp}$.

(b) State and prove characterization of the Hyperspace in a n.l. Space.

(c) State and prove closed graph theorem.